Kondo Effect in the Presence of Spin-Orbit Coupling

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Recently, a series of noncentrosymmetric superconductors has been a subject of considerable interest since the discovery of superconductivity in $CePt_3Si$. In noncentrosymmetric materials, the degeneracy of bands is lifted in the presence of spin-orbit coupling. This will bring about new effects in the Kondo effect since the band degeneracy plays an important role in the scattering of electrons by localized spins. We investigate the single-impurity Kondo problem in the presence of spin-orbit coupling. We examine the effect of spin-orbit coupling on the scattering of conduction electrons, by using the Green's function method, for the s-d Hamiltonian, with employing a decoupling procedure. As a result, we obtain a closed system of equations of Green's functions, from which we can calculate physical quantities. The Kondo temperature T_K is estimated from a singularity of Green's functions. We show that T_K is reduced as the spin-orbit coupling constant α is increased. When $2\alpha k_F$ is comparable to or greater than $k_BT_K(\alpha = 0)$, T_K shows an abrupt decrease as a result of the band splitting. This suggests a Kondo collapse accompanied with a sharp decrease of T_K. The $\log T$-dependence of the resistivity will be concealed by the spin-orbit interaction.

I. INTRODUCTION

The Kondo effect has attracted many researchers since the discovery of the solution of the resistance minimum[1, 2]. The effect arises from the interactions between a single magnetic atom and the many electrons in a metal. Metals, when magnetic atoms are added, and rare earth compounds exhibit many interesting phenomena that are related to the Kondo effect. The spin-flip scattering of a conduction-electron spin by a localized impurity spin gives rise to a term proportional to $\ln T$ in the resistivity.

Superconductors without inversion symmetry have attracted much attention since the discovery of superconductivity in $CePt_3Si$[3]. A group of noncentrosymmetric rare-earth compounds has been reported to exhibit superconductivity: for example, Li_2Pt_3B[4, 5], $CeIrSi_3$, $CeCoGe_3$, $CeIrGe_3$[6–8], and $LaNiC_2$[9]. The absence of spatial inversion yields the splitting of bands due to a spin-orbit interaction[10, 11].

The influence of the spin-orbit interaction was discussed very recently in two-dimensional systems starting from the single-impurity Anderson model[12–14]. In the conventional Kondo problem, the conduction-electron states with spin up and down are degenerate. We expect that the band splitting has a large effect on the Kondo effect, and is closely related to a multi-channel Kondo problem. The purpose of this paper is to investigate this subject on the basis of the s-d Hamiltonian with the spin-orbit interaction of Rashba type in three dimensions at finite temperature. We calculate Green's functions and evaluate the Kondo temperature T_K from a singularity of them. We show that T_K is reduced as a result of the band splitting and shows a abrupt decrease when αk_F is comparable to k_BT_K.

The paper is organized as follows. In Section II we show the Hamiltonian, and in Section III we derive equations for Green's functions. We obtain an approximate solution in Section IV. The Kondo temperature and correction to resistivity are discussed in subsequent Sections V and VI. In Section VII we examine the strong limit of the spin-orbit interaction where the details of calcuations are shown in Appexdix.

II. MODEL HAMILTONIAN

The Hamiltonian is $H = H_0 + H_{sd} = H_K + H_{so} + H_{sd}$ where

$$H_K = \sum_{\mathbf{k}} \xi_{\mathbf{k}} (c^\dagger_{\mathbf{k}\uparrow} c_{\mathbf{k}\uparrow} + c^\dagger_{\mathbf{k}\downarrow} c_{\mathbf{k}\downarrow}), \tag{1}$$

$$H_{so} = \sum_{\mathbf{k}} [\alpha(ik_x + k_y) c^\dagger_{\mathbf{k}\uparrow} c_{\mathbf{k}\downarrow} + \alpha(-ik_x + k_y) c^\dagger_{\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow}], \tag{2}$$

$$H_{sd} = -\frac{J}{2}\frac{1}{N}\sum_{\mathbf{k}\mathbf{k}'} [S_z(c^\dagger_{\mathbf{k}\uparrow} c_{\mathbf{k}'\uparrow} - c^\dagger_{\mathbf{k}\downarrow} c_{\mathbf{k}'\downarrow}) + S_+ c^\dagger_{\mathbf{k}\downarrow} c_{\mathbf{k}'\uparrow} + S_- c^\dagger_{\mathbf{k}\uparrow} c_{\mathbf{k}'\downarrow}]. \tag{3}$$

$\xi_{\mathbf{k}}$ is defined by $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$ where $\epsilon_{\mathbf{k}}$ is the dispersion relation of the conduction electrons and μ is the chemical potential. $c_{\mathbf{k}\sigma}$ and $c^\dagger_{\mathbf{k}\sigma}$ are annihilation and creation operators, respectively. We set $H_0 = H_K + H_{so}$. S_+, S_- and S_z denote the operators of the localized spin. We consider the spin-orbit interaction of Rashba type in H_{so}. α indicates the coupling constant of the spin-orbit interaction. The term H_{sd} indicates the s-d interaction between the conduction electrons and the localized spin, with the coupling constant J. J is negative for the antiferromagnetic interaction.

III. GREEN'S FUNCTIONS

First, we define Green's functions of the conduction electrons

$$\begin{aligned} G_{\mathbf{k}\mathbf{k}'\sigma}(\tau) &= -\langle T_\tau c_{\mathbf{k}\sigma}(\tau) c^\dagger_{\mathbf{k}'\sigma}(0)\rangle, \quad (4)\\ F_{\mathbf{k}\mathbf{k}'}(\tau) &= -\langle T_\tau c_{\mathbf{k}\downarrow}(\tau) c^\dagger_{\mathbf{k}'\uparrow}(0)\rangle, \quad (5)\end{aligned}$$

where T_τ is the time ordering operator. We note that the spin operators satisfy the following relations:

$$\begin{aligned} S_\pm S_z &= \mp\frac{1}{2}S_z, \quad S_z S_\pm = \pm\frac{1}{2}S_\pm, \quad (6)\\ S_+S_- &= \frac{3}{4} + S_z - S_z^2, \quad (7)\\ S_-S_+ &= \frac{3}{4} - S_z - S_z^2. \quad (8)\end{aligned}$$

We also define Green's functions which include the localized spins as well as the conduction electron operators. They are for example, following the notation of Zubarev[15],

$$\begin{aligned} \langle\langle S_z c_{\mathbf{k}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_\tau &= -\langle T_\tau S_z c_{\mathbf{k}\uparrow}(\tau) c^\dagger_{\mathbf{k}'\uparrow}(0)\rangle, \quad (9)\\ \langle\langle S_- c_{\mathbf{k}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_\tau &= -\langle T_\tau S_- c_{\mathbf{k}\downarrow}(\tau) c^\dagger_{\mathbf{k}'\uparrow}(0)\rangle, \quad (10)\\ \langle\langle S_z c_{\mathbf{k}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_\tau &= -\langle T_\tau S_z c_{\mathbf{k}\downarrow}(\tau) c^\dagger_{\mathbf{k}'\uparrow}(0)\rangle, \quad (11)\\ \langle\langle S_- c_{\mathbf{k}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_\tau &= -\langle T_\tau S_- c_{\mathbf{k}\uparrow}(\tau) c^\dagger_{\mathbf{k}'\uparrow}(0)\rangle. \quad (12)\end{aligned}$$

The Fourier transforms are defined as usual:

$$\begin{aligned} G_{\mathbf{k}\mathbf{k}'\sigma}(\tau) &= \frac{1}{\beta}\sum_n e^{-i\omega_n\tau} G_{\mathbf{k}\mathbf{k}'\sigma}(i\omega_n), \quad (13)\\ F_{\mathbf{k}\mathbf{k}'}(\tau) &= \frac{1}{\beta}\sum_n e^{-i\omega_n\tau} F_{\mathbf{k}\mathbf{k}'}(i\omega_n), \quad (14)\\ \langle\langle S_z c_{\mathbf{k}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_\tau &= \frac{1}{\beta}\sum_n e^{-i\omega_n\tau}\langle\langle S_z c_{\mathbf{k}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega_n}, \quad (15)\\ &\cdots\cdots. \quad (16)\end{aligned}$$

From the commutation relations

$$\begin{aligned} [H_0, c_{\mathbf{k}\uparrow}] &= -\xi_{\mathbf{k}} c_{\mathbf{k}\uparrow} - \alpha(ik_x + k_y) c_{\mathbf{k}\downarrow}, \quad (17)\\ [H_{sd}, c_{\mathbf{k}\uparrow}] &= -\frac{J}{2N}\sum_{\mathbf{k}'}(-S_z c_{\mathbf{k}'\uparrow} - S_- c_{\mathbf{k}'\downarrow}), \quad (18)\end{aligned}$$

the equation of motion for $G_{\mathbf{k}\mathbf{k}'\uparrow}(\tau)$ reads

$$\begin{aligned} \frac{\partial}{\partial\tau}G_{\mathbf{k}\mathbf{k}'\uparrow}(\tau) &= -\delta(\tau)\delta_{\mathbf{k}\mathbf{k}'} - \xi_{\mathbf{k}} G_{\mathbf{k}\mathbf{k}'\uparrow}(\tau)\\ &\quad -\alpha(ik_x + k_y) F_{\mathbf{k}\mathbf{k}'}(\tau)\\ &\quad + \frac{J}{2N}\sum_{\mathbf{q}}[\langle\langle S_z c_{\mathbf{q}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_\tau + \langle\langle S_- c_{q\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_\tau]. \quad (19)\end{aligned}$$

Similarly, the equation of motion for $F_{\mathbf{k}\mathbf{k}'}$ is

$$\begin{aligned} \frac{\partial}{\partial\tau}F_{\mathbf{k}\mathbf{k}'}(\tau) &= -\xi_{\mathbf{k}} F_{\mathbf{k}\mathbf{k}'} - \alpha(-ik_x + k_y) G_{\mathbf{k}\mathbf{k}'\uparrow}(\tau)\\ &\quad - \frac{J}{2N}\sum_q[\langle\langle S_z c_{\mathbf{q}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_\tau - \langle\langle S_+ c_{\mathbf{q}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_\tau]. \quad (20)\end{aligned}$$

We define

$$\begin{aligned} \Gamma_{\mathbf{k}\mathbf{k}'}(\tau) &= \frac{1}{\beta}\sum_n e^{-i\omega_n}\Gamma_{\mathbf{k}\mathbf{k}'}(i\omega_n)\\ &= \langle\langle S_z c_{\mathbf{k}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_\tau + \langle\langle S_- c_{\mathbf{k}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_\tau, \quad (21)\\ \Phi_{\mathbf{q}\mathbf{k}}(\tau) &= \frac{1}{\beta}\sum_n e^{-i\omega_n\tau}\Phi_{\mathbf{q}\mathbf{k}}(i\omega_n)\\ &= \langle\langle S_z c_{\mathbf{q}\downarrow} - S_+ c_{\mathbf{q}\uparrow}; c^\dagger_{\mathbf{k}\uparrow}\rangle\rangle, \quad (22)\end{aligned}$$

then we obtain

$$\begin{aligned} (i\omega_n - \xi_{\mathbf{k}})G_{\mathbf{k}\mathbf{k}'\uparrow}(i\omega_n) &= \delta_{\mathbf{k}\mathbf{k}'} + \alpha(ik_x + k_y)F_{\mathbf{k}\mathbf{k}'}(i\omega_n)\\ &\quad - \frac{J}{2N}\sum_{\mathbf{q}\mathbf{k}'}\Gamma_{\mathbf{q}\mathbf{k}'}(i\omega_n), \quad (23)\\ (i\omega_n - \xi_{\mathbf{k}})F_{\mathbf{k}\mathbf{k}'}(i\omega_n) &= \alpha(-ik_x + k_y)G_{\mathbf{k}\mathbf{k}'\uparrow}(i\omega_n)\\ &\quad + \frac{J}{2N}\sum_q\Phi_{\mathbf{q}\mathbf{k}'}(i\omega_n). \quad (24)\end{aligned}$$

To obtain the solution to the equations above, we need Green's functions in eqs.(9)-(12). The equations of motion for $\langle\langle S_z c_{\mathbf{k}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle$ and $\langle\langle S_- c_{\mathbf{k}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle$ are

$$\begin{aligned} &(i\omega - \xi_{\mathbf{k}})\langle\langle S_z c_{\mathbf{k}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}\\ &= \langle S_z\rangle\delta_{\mathbf{k}\mathbf{k}'} + \alpha(ik_x + k_y)\langle\langle S_z c_{\mathbf{k}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}\\ &- \frac{J}{2N}\sum_{\mathbf{q}}\left[\langle\langle S_z^2 c_{\mathbf{q}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} + \frac{1}{2}\langle\langle S_- c_{\mathbf{q}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}\right]\\ &- \frac{J}{2N}\sum_{\mathbf{q}\mathbf{q}'}\Big[\langle\langle S_+ c_{\mathbf{k}\uparrow} c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{q}'\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}\\ &- \langle\langle S_- c_{\mathbf{k}\uparrow} c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{q}'\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}\Big], \quad (25)\end{aligned}$$

$$\begin{aligned} &(i\omega - \xi_{\mathbf{k}})\langle\langle S_- c_{\mathbf{k}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}\\ &= \alpha(-ik_x + k_y)\langle\langle S_- c_{\mathbf{k}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}\\ &- \frac{J}{4N}\sum_{\mathbf{q}\mathbf{q}'}\langle\langle S_- c_{\mathbf{q}'\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}\\ &+ \frac{J}{2N}\sum_{\mathbf{q}\mathbf{q}'}\Big[\langle\langle S_- c_{\mathbf{k}\downarrow} c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{q}'\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}\\ &-\langle\langle S_- c_{\mathbf{k}\downarrow} c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{q}'\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} - 2\langle\langle S_z c_{\mathbf{k}\downarrow} c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{q}'\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}\Big]\\ &-\frac{J}{2N}\sum_{\mathbf{q}'}\langle\langle S_+ S_- c_{\mathbf{q}'\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}. \quad (26)\end{aligned}$$

We use the commutation relation $S_+S_- = 3/4 + S_z - S_z^2$ to obtain

$$
\begin{aligned}
&(i\omega - \xi_{\mathbf{k}})\Gamma_{\mathbf{k}\mathbf{k}'}(i\omega) \\
=\;& \delta_{\mathbf{k}\mathbf{k}'}\langle S_z\rangle + \alpha(ik_x + k_y)\langle\langle S_z c_{\mathbf{k}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} \\
+\;& \alpha(-ik_x + k_y)\langle\langle S_- c_{\mathbf{k}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} \\
-\;& \frac{J}{2N}\sum_{\mathbf{q}}\left[\frac{3}{4}\langle\langle c_{\mathbf{q}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} + \Gamma_{\mathbf{q}\mathbf{k}'}(i\omega)\right] \\
-\;& \frac{J}{2N}\sum_{\mathbf{q}\mathbf{q}'}\Big[\langle\langle S_+ c_{\mathbf{k}\uparrow} c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{q}'\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} \\
-\;& \langle\langle S_- c_{\mathbf{k}\uparrow} c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{q}'\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} - \langle\langle S_- c_{\mathbf{k}\downarrow} c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{q}'\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} \\
+\;& \langle\langle S_- c_{\mathbf{k}\downarrow} c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{q}'\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} \\
+\;& 2\langle\langle S_z c_{\mathbf{k}\downarrow} c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{q}'\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}\Big]. \qquad (27)
\end{aligned}
$$

Here we assume that $\langle S_z\rangle = 0$. Now we need $\langle\langle S_z c_{\mathbf{k}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle$ and $\langle\langle S_- c_{\mathbf{k}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle$ to obtain a solution for $\Gamma_{\mathbf{k}\mathbf{k}'}$. The equations for $\langle\langle S_z c_{\mathbf{k}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle$ and $\langle\langle S_- c_{\mathbf{k}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle$ read

$$
\begin{aligned}
&(i\omega - \xi_{\mathbf{k}})\langle\langle S_z c_{\mathbf{k}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} \\
=\;& \alpha(-ik_x + k_y)\langle\langle S_z c_{\mathbf{k}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} \\
+\;& \frac{J}{2N}\sum_{q}\left[\langle\langle S_z^2 c_{\mathbf{q}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} - \frac{1}{2}\langle\langle S_+ c_{\mathbf{q}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}\right] \\
-\;& \frac{J}{2N}\sum_{\mathbf{q}\mathbf{q}'}\Big[\langle\langle S_+ c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{q}'\downarrow} c_{\mathbf{k}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} \\
-\;& \langle\langle S_- c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{q}'\downarrow} c_{\mathbf{k}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}\Big], \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
&(i\omega - \xi_{\mathbf{k}})\langle\langle S_- c_{\mathbf{k}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} \\
=\;& -\delta_{\mathbf{k}\mathbf{k}'}\langle S_-\rangle + \alpha(ik_x + k_y)\langle\langle S_- c_{\mathbf{k}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} \\
+\;& \frac{J}{4N}\sum_{\mathbf{q}'}\langle\langle S_- c_{\mathbf{q}'\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} \\
+\;& \frac{J}{2N}\sum_{\mathbf{q}\mathbf{q}'}\Big[\langle\langle S_- c_{\mathbf{k}\uparrow} c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{q}'\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} \\
-\;& \langle\langle S_- c_{\mathbf{k}\uparrow} c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{q}'\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} + 2\langle\langle S_z c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{q}'\downarrow} c_{\mathbf{k}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}\Big]. \qquad (29)
\end{aligned}
$$

IV. APPROXIMATE SOLUTION

We assume that the spin-orbit coupling α is small and we keep terms up to the order of α. We adopt the approximation that

$$
\langle\langle S_z c_{\mathbf{k}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} = \frac{\alpha(-ik_x + k_y)}{i\omega - \xi_{\mathbf{k}}}\langle\langle S_z c_{\mathbf{k}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}, \qquad (30)
$$

$$
\langle\langle S_- c_{\mathbf{k}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} = \frac{\alpha(ik_x + k_y)}{i\omega - \xi_{\mathbf{k}}}\langle\langle S_- c_{\mathbf{k}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}. \qquad (31)
$$

This means that we have neglected the terms of the order of $J\alpha$ in the right-hand side. Then we obtain

$$
\begin{aligned}
&\left(i\omega - \xi_{\mathbf{k}} - \frac{\alpha^2(k_x^2 + k_y^2)}{i\omega - \xi_{\mathbf{k}}}\right)\Gamma_{\mathbf{k}\mathbf{k}'}(i\omega) \\
=\;& -\frac{J}{2N}\sum_{\mathbf{q}}\left[\frac{3}{4}\langle\langle c_{\mathbf{q}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} + \Gamma_{\mathbf{q}\mathbf{k}'}(i\omega)\right] \\
-\;& \frac{J}{2N}\sum_{\mathbf{q}\mathbf{q}'}\Big[\langle\langle S_+ c_{\mathbf{k}\uparrow} c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{q}'\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} \\
-\;& \langle\langle S_- c_{\mathbf{k}\uparrow} c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{q}'\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} - \langle\langle S_- c_{\mathbf{k}\downarrow} c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{q}'\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} \\
+\;& \langle\langle S_- c_{\mathbf{k}\downarrow} c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{q}'\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} + 2\langle\langle S_z c_{\mathbf{k}\downarrow} c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{q}'\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}\Big] \qquad (32)
\end{aligned}
$$

We use the same approximation in the right-hand side of eq.(23), that is, $(i\omega - \xi_{\mathbf{k}})F_{\mathbf{k}\mathbf{k}'}(i\omega) = \alpha(-ik_x + k_y)G_{\mathbf{k}\mathbf{k}'\uparrow}(i\omega)$, and we have

$$
\begin{aligned}
(i\omega - \xi_{\mathbf{k}})G_{\mathbf{k}\mathbf{k}'\uparrow}(i\omega) &= \delta_{\mathbf{k}\mathbf{k}'} - \frac{J}{2N}\sum_{\mathbf{p}}\Gamma_{\mathbf{p}\mathbf{k}'}(i\omega) \\
&+ \frac{\alpha^2(k_x^2 + k_y^2)}{i\omega - \xi_{\mathbf{k}}}G_{\mathbf{k}\mathbf{k}'\uparrow}(i\omega) \qquad (33)
\end{aligned}
$$

Here, we employ the decoupling approximation procedure for Green's functions[16, 17]. Many-body Green's functions are approximated as follows.

$$
\begin{aligned}
\langle\langle S_- c_{\mathbf{k}\uparrow} c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{q}'\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} &\approx \langle c_{\mathbf{k}\uparrow} c^\dagger_{\mathbf{q}\uparrow}\rangle\langle\langle S_- c_{\mathbf{q}'\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} \\
&+ \langle S_- c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{q}'\downarrow}\rangle\langle\langle c_{\mathbf{k}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}, \qquad (34)
\end{aligned}
$$

$$
\begin{aligned}
\langle\langle S_+ c_{\mathbf{k}\uparrow} c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{q}'\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} &\approx \langle S_+ c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{q}'\uparrow}\rangle\langle\langle c_{\mathbf{k}\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} \\
&- \langle S_+ c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{k}'\uparrow}\rangle\langle\langle c_{\mathbf{q}'\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}, \qquad (35)
\end{aligned}
$$

$$
\begin{aligned}
\langle\langle S_z c_{\mathbf{k}\downarrow} c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{q}'\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} &\approx \langle c_{\mathbf{k}\downarrow} c^\dagger_{\mathbf{q}\downarrow}\rangle\langle\langle S_z c_{\mathbf{q}'\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} \\
&- \langle S_z c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{k}\downarrow}\rangle\langle\langle c_{\mathbf{q}'\uparrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}, \qquad (36)
\end{aligned}
$$

$$
\begin{aligned}
\langle\langle S_- c_{\mathbf{k}\downarrow} c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{q}'\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} &\approx \langle c_{\mathbf{q}\downarrow} c^\dagger_{\mathbf{q}'\downarrow}\rangle\langle\langle S_- c_{\mathbf{k}\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega} \\
&+ \langle c_{\mathbf{k}\downarrow} c^\dagger_{\mathbf{q}\downarrow}\rangle\langle\langle S_- c_{\mathbf{q}'\downarrow}; c^\dagger_{\mathbf{k}'\uparrow}\rangle\rangle_{i\omega}. \qquad (37)
\end{aligned}
$$

We define

$$n_{\mathbf{k}} = \sum_{\mathbf{q}} \langle c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{k}\uparrow}\rangle = \sum_{\mathbf{q}} \langle c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{k}\downarrow}\rangle, \quad (38)$$

$$m_k = 3\sum_{\mathbf{q}} \langle S_- c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{k}\downarrow}\rangle = 2\sum_{\mathbf{q}} (\langle S_z c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{k}\uparrow}\rangle + \langle S_- c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{k}\downarrow}\rangle). \quad (39)$$

We used the relation obtained from the rotational symmetry in the spin space,

$$\langle S_- c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{q}'\downarrow}\rangle = \langle S_+ c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{q}'\uparrow}\rangle = 2\langle S_z c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{q}'\uparrow}\rangle = -2\langle S_z c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{q}'\downarrow}\rangle. \quad (40)$$

Then, after the analytic continuation $i\omega \to \omega + i\delta$, we have

$$\left(\omega - \xi_{\mathbf{k}} - \frac{\alpha^2 k_\perp^2}{\omega - \xi_{\mathbf{k}}}\right)\Gamma_{\mathbf{k}\mathbf{k}'}(\omega) + \left(\frac{3}{4} - m_{\mathbf{k}}\right) \times \frac{J}{2N}\sum_{\mathbf{q}} G_{\mathbf{q}\mathbf{k}'}(\omega) + \left(n_{\mathbf{k}} - \frac{1}{2}\right)\frac{J}{N}\sum_{\mathbf{q}} \Gamma_{\mathbf{q}\mathbf{k}'}(\omega) = 0, \quad (41)$$

$$\left(\omega - \xi_{\mathbf{k}} - \frac{\alpha^2 k_\perp^2}{\omega - \xi_{\mathbf{k}}}\right) G_{\mathbf{k}\mathbf{k}'}(\omega) + \frac{J}{2N}\sum_{\mathbf{q}} \Gamma_{\mathbf{q}\mathbf{k}'}(\omega) = \delta_{\mathbf{k}\mathbf{k}'}, \quad (42)$$

where we set $k_\perp^2 = k_x^2 + k_y^2$. Then, we obtain from eqs.(41) and (42) that

$$\begin{aligned}\Gamma_{\mathbf{k}\mathbf{k}'}(\omega) &= G^0_{\mathbf{k}}(\omega)\left(m_{\mathbf{k}} - \frac{3}{4}\right)\frac{J}{2N} G^0_{\mathbf{k}'}(\omega) \\ &- G^0_{\mathbf{k}}(\omega)\left[\left(n_{\mathbf{k}} - \frac{1}{2}\right) J + \left(m_{\mathbf{k}} - \frac{3}{4}\right)\left(\frac{J}{2}\right)^2 F(\omega)\right] \\ &\times \frac{1}{N}\sum_{\mathbf{q}} \Gamma_{\mathbf{q}\mathbf{k}}(\omega),\end{aligned} \quad (43)$$

where

$$G^0_{\mathbf{k}}(\omega) = \frac{1}{2}\left(\frac{1}{\omega - \xi_{\mathbf{k}} + \alpha k_\perp} + \frac{1}{\omega - \xi_{\mathbf{k}} - \alpha k_\perp}\right) \quad (44)$$

$$F(\omega) = \frac{1}{N}\sum_{\mathbf{k}} G^0_{\mathbf{k}}(\omega), \quad (45)$$

$$G(\omega) = \frac{1}{N}\sum_{\mathbf{k}} \left(n_{\mathbf{k}} - \frac{1}{2}\right) G^0_{\mathbf{k}}(\omega), \quad (46)$$

$$\Gamma(\omega) = \frac{1}{N}\sum_{\mathbf{k}} \left(m_{\mathbf{k}} - \frac{3}{4}\right) G^0_{\mathbf{k}}(\omega). \quad (47)$$

Because of

$$\frac{1}{N}\sum_{\mathbf{q}} \Gamma_{\mathbf{q}\mathbf{k}}(\omega) = \frac{J}{2N}\Gamma(\omega) G^0_{\mathbf{k}}(\omega)\frac{1}{1 + JG(\omega) + (J/2)^2\Gamma(\omega)F(\omega)}, \quad (48)$$

we obtain

$$\begin{aligned}\Gamma_{\mathbf{k}\mathbf{k}'}(\omega) &= \frac{J}{2N} G^0_{\mathbf{k}}(\omega) G^0_{\mathbf{k}'}(\omega)\Big[\left(m_{\mathbf{k}} - \frac{3}{4}\right)(1 + JG(\omega)) \\ &- \left(n_{\mathbf{k}} - \frac{1}{2}\right) J\Gamma(\omega)\Big] \\ &\times \frac{1}{1 + JG(\omega) + (J/2)^2\Gamma(\omega)F(\omega)},\end{aligned} \quad (49)$$

$$\begin{aligned}G_{\mathbf{k}\mathbf{k}'}(\omega) &= \delta_{\mathbf{k}\mathbf{k}'} G^0_{\mathbf{k}}(\omega) - \frac{J^2}{4N}\Gamma(\omega) G^0_{\mathbf{k}}(\omega) G^0_{\mathbf{k}'}(\omega) \\ &\times \frac{1}{1 + JG(\omega) + (J/2)^2\Gamma(\omega)F(\omega)} \\ &= \delta_{\mathbf{k}\mathbf{k}'} G^0_{\mathbf{k}}(\omega) + \frac{J}{N} G^0_{\mathbf{k}}(\omega) G^0_{\mathbf{k}'}(\omega) t(\omega),\end{aligned} \quad (50)$$

where we defined

$$t(\omega) = -\frac{J}{4}\frac{\Gamma(\omega)}{1 + JG(\omega) + (J/2)^2\Gamma(\omega)F(\omega)}. \quad (51)$$

$m_{\mathbf{k}}$ is given by

$$\begin{aligned}m^*_{\mathbf{k}} &= 2\sum_{\mathbf{q}} (\langle S_z c^\dagger_{\mathbf{k}\uparrow} c_{\mathbf{q}\uparrow}\rangle \langle S_- c^\dagger_{\mathbf{k}\uparrow} c_{\mathbf{q}\downarrow}\rangle) = 2\sum_{\mathbf{q}} \Gamma_{\mathbf{q}\mathbf{k}}(\tau = -0) \\ &= \frac{2}{\beta}\sum_{\mathbf{q}\omega_n} e^{i\omega_n\delta}\Gamma_{\mathbf{q}\mathbf{k}}(i\omega_n),\end{aligned} \quad (52)$$

where δ is an infinitesimal constant. Because $m_{\mathbf{k}}$ is real, we obtain

$$m_{\mathbf{k}} = -4\frac{1}{\beta}\sum_{\omega_n} e^{i\omega_n\delta} G^0_{\mathbf{k}}(i\omega_n) t(i\omega_n). \quad (53)$$

Similarly we have

$$n_{\mathbf{k}} = \frac{1}{\beta}\sum_{\omega_n} G^0_{\mathbf{k}}(i\omega_n)(1 + JF(i\omega_n)t(i\omega_n)). \quad (54)$$

V. KONDO TEMPERATURE

A singularity of $t(\omega)$ determines the characteristic temperature of the system. We investigate the high-temperature region where $m_{\mathbf{k}} = 0$. Then,

$$\Gamma(\omega) = -\frac{3}{4}F(\omega). \quad (55)$$

We obtain for $\mathbf{k} = \mathbf{k}'$

$$G_{\mathbf{k}\mathbf{k}}(\omega)^{-1} = G^0_{\mathbf{k}}(\omega)^{-1} - \frac{3J^2}{16N}\frac{F(\omega)}{1 + JG(\omega)} + O(J^4). \quad (56)$$

The Kondo temperature T_K is determined by the vanishing of the denominator in this equation:

$$1 - J\frac{1}{2N}\sum_{\mathbf{k}}\left(\frac{1}{\omega - \xi_{\mathbf{k}} + \alpha k_\perp} + \frac{1}{\omega - \xi_{\mathbf{k}} - \alpha k_\perp}\right) \times \frac{1}{4}\left(\tanh\left(\frac{\xi_{\mathbf{k}} - \alpha k_\perp}{2T_K}\right) + \tanh\left(\frac{\xi_{\mathbf{k}} + \alpha k_\perp}{2T_K}\right)\right) = 0, \quad (57)$$

where

$$\xi_{\mathbf{k}} = \frac{1}{2m}(k_\perp^2 + k_z^2) - \mu. \tag{58}$$

We have used $n_{\mathbf{k}} = (f(\xi_k - \alpha k_\perp) + f(\xi_k + \alpha k_\perp))/2$ by neglecting the term of the order of J^2. By using the expansion,

$$\tanh\left(\frac{z}{2}\right) = \sum_{n=-\infty}^{\infty} \frac{1}{z - i\pi(2n+1)}, \tag{59}$$

the equation for T_K is

$$\begin{aligned} 1 &= J \sum_{n=-\infty}^{\infty} \frac{1}{8(2\pi)^2}\sqrt{\frac{2m}{\mu}} \int_0^K dk_\perp k_\perp \Big[2\frac{i\pi T_K \mathrm{sign}(2n+1)}{\omega - i\pi(2n+1)T_K} \\ &+ \frac{i\pi T_K \mathrm{sign}(2n+1)}{\omega + 2\alpha k_\perp - i\pi(2n+1)T_K} \\ &+ \frac{i\pi T_K \mathrm{sign}(2n+1)}{\omega - 2\alpha k_\perp - i\pi(2n+1)T_K} \Big], \end{aligned} \tag{60}$$

where K is a cutoff and we use an approximate expression

$$\begin{aligned} &\int_{-K}^{K} dk_z \frac{1}{\omega - k_\perp^2/(2m) + \alpha k_\perp - k_z^2/(2m) + \mu} \\ &\times \frac{1}{k_\perp^2/(2m) + k_z^2/(2m) + \alpha k_\perp - \mu - i\pi(2n+1)T_K} \\ &\approx \sqrt{\frac{2m}{\mu}} i\pi \mathrm{sign}(2n+1) T_K \frac{1}{\omega + 2\alpha k_\perp - i\pi(2n+1)T_K}. \end{aligned} \tag{61}$$

We set an cutoff $n_0 \equiv D/(2\pi T_K)$ in the summation with respect to n. By using the formula for the digamma function,

$$\sum_{n=0}^{n_0} \frac{1}{n + \frac{1}{2} + x} = \psi\left(\frac{1}{2} + x + n_0\right) - \psi\left(\frac{1}{2} + x\right), \tag{62}$$

we obtain

$$\begin{aligned} 1 &= |J| \frac{1}{32\pi^2}\sqrt{\frac{2m}{\mu}} \int_0^K dk_\perp k_\perp \Big[4\log\left(\frac{2e^\gamma D}{\pi T_K}\right) + 2\psi\left(\frac{1}{2}\right) \\ &- \frac{1}{2}\psi\left(\frac{1}{2} - \frac{\omega + 2\alpha k_\perp}{i2\pi T_K}\right) \\ &- \frac{1}{2}\psi\left(\frac{1}{2} + \frac{\omega + 2\alpha k_\perp}{i2\pi T_K}\right) - \frac{1}{2}\psi\left(\frac{1}{2} - \frac{\omega - 2\alpha k_\perp}{i2\pi T_K}\right) \\ &- \frac{1}{2}\psi\left(\frac{1}{2} + \frac{\omega - 2\alpha k_\perp}{i2\pi T_K}\right) \Big]. \end{aligned} \tag{63}$$

We set $\mu = k_F^2/(2m)$ and $K = 2k_F$, and expand the digamma function in terms of $\alpha k_\perp/(2\pi T_K)$. For $\omega = 0$, we have

$$\begin{aligned} 1 &= |J| \frac{m k_F}{2\pi^2} \Big[\log\left(\frac{2e^\gamma D}{\pi T_K}\right) - \frac{7\zeta(3)}{2\pi^2}\left(\frac{2\alpha k_F}{T_K}\right)^2 \\ &+ \frac{31\zeta(5)}{12\pi^4}\left(\frac{2\alpha k_F}{T_K}\right)^4 - \dots \Big]. \end{aligned} \tag{64}$$

This yields the temperature T_K as

$$\begin{aligned} k_B T_K &= \frac{2e^\gamma D}{\pi} \exp\Big[-\frac{1}{\rho_F |J|} - \frac{7\zeta(3)}{2\pi^2}\left(\frac{2\alpha k_F}{k_B T_K}\right)^2 \\ &+ \frac{31\zeta(5)}{12\pi^4}\left(\frac{2\alpha k_F}{k_B T_K}\right)^4 - \cdots \Big], \end{aligned} \tag{65}$$

where we introduced the Boltzmann constant k_B and the density of states $\rho_F = m k_F/(2\pi^2)$. This is a self-consistency equation for T_K, and yields

$$\begin{aligned} x &= \exp\Big[-0.21314\left(\frac{\alpha_r}{x}\right)^2 + 0.0550\left(\frac{\alpha_r}{x}\right)^4 \\ &- 0.01655\left(\frac{\alpha_r}{x}\right)^6 + 0.005396\left(\frac{\alpha_r}{x}\right)^8 - 0.001822\left(\frac{\alpha_r}{x}\right)^{10} \\ &+ \cdots \Big] \\ &\equiv g(x), \end{aligned} \tag{66}$$

with variables

$$x = T_K/T_K^0, \quad \alpha_r = 2\alpha k_F/k_B T_K^0, \tag{67}$$

where

$$k_B T_K^0 = \frac{2e^\gamma D}{\pi} \exp\left(-\frac{1}{\rho_F |J|}\right). \tag{68}$$

We expanded $g(x)$ in powers of α_r/x up to the tenth order. The function $g(x)$ is shown in Fig.1, where higher-order terms are small and negligible except near $x \sim 0$. The equation $x = g(x)$ has no finite solution when $\alpha_r > 1.045$. This indicates that T_K vanishes when the spin-orbit coupling αk_F is greater than $1.045 k_B T_K^0$, and indicates a Kondo collapse with a sharp decrease of T_K. This may overestimate the reduction of T_K. When α is very large, if we use the asymptotic relation $\psi(1/2+z) \sim \log(z)$, we obtain

$$1 \simeq \rho_F |J| \Big[\log\left(\frac{2e^\gamma D}{\pi k_B T}\right) - \frac{1}{2}\log\left(\frac{2\alpha k_F}{\pi k_B T}\right) + \frac{1}{4} \Big]. \tag{69}$$

This yields

$$k_B T_K \simeq \frac{\sqrt{e}}{2\alpha k_F} \frac{(2e^\gamma D)^2}{\pi} \exp\left(-\frac{2}{\rho_F |J|}\right) = \frac{\pi\sqrt{e}}{\alpha_r} k_B T_K^0, \tag{70}$$

for $\alpha_r \gg 1$. We show T_K as a function of α_r in Fig.2.

We expect that,in the strong coupling limit $\alpha_r \gg 1$, T_K should approach that of single-band model:

$$k_B T_K^\alpha = \frac{2e^\gamma D}{\pi} \exp\left(-\frac{2}{\rho_F |J|}\right). \tag{71}$$

We will show this in the section 7. This agrees with eq.(70) for $\alpha k_F \sim D$. This is very small compared to the original T_K because $T_K^\alpha/T_K^0 \simeq k_B T_K^0/D$. Therefore T_K decreases as α_r is increased and shows up a sharp decrease at $\alpha_r \sim 1$.

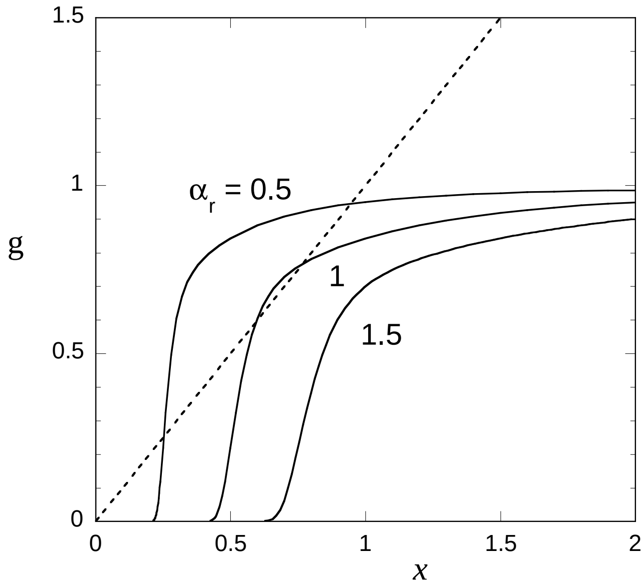

FIG. 1: $g(x) = \exp(-(7\zeta(3)/2\pi^2)(\alpha_r/x)^2 + \cdots)$ up to the tenth order of α_r/x as a function of x for $\alpha_r = 0.5$, 1 and 1.5. The straight line is a linear function x.

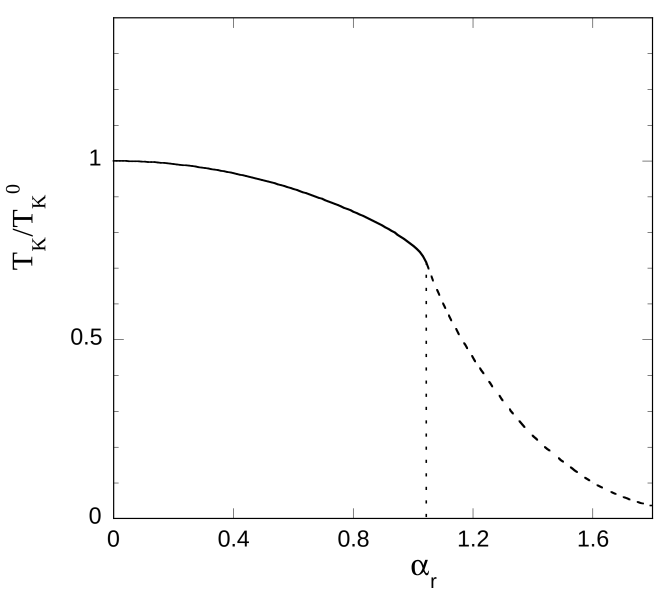

FIG. 2: T_K as a function of α_r. $x = g(x)$ has no solution for $\alpha_r > 1.045$ (Kondo collapse) as indicated by dotted line. The dashed line is an expected line.

VI. CORRECTION TO RESISTIVITY

The imaginary part of $G_{\mathbf{kk}}(\omega)^{-1}$ gives the scattering rate of conduction electrons due to the localized spin. The inverse of the life time $\tau_k(\omega)$ is

$$\begin{aligned}\frac{1}{\tau_k(\omega)} &= n_i N \mathrm{Im} G_{\mathbf{kk}}(\omega)^{-1} \\ &= \frac{3n_i J^2}{16}\pi\rho^\alpha(\omega)\frac{1+JK(\omega)}{(1+JK(\omega))^2+(JL(\omega))^2},\end{aligned} \tag{72}$$

where n_i is the concentration of magnetic impurities. We have defined $K(\omega) = \mathrm{Re}G(\omega+i\delta)$, $L(\omega) = -\mathrm{Im}G(\omega+i\delta)$, and $\rho^\alpha(\omega) = -(1/\pi)\mathrm{Im}F(\omega+i\delta)$. Because we obtain

$$K(0) \simeq \rho_F\Big[\log\left(\frac{2e^\gamma D}{\pi k_B T}\right) - \frac{7\zeta(3)}{4\pi^2}\left(\frac{2\alpha k_F}{k_B T}\right)^2\Big], \tag{73}$$

the formula of the conductivity yields

$$\begin{aligned}\sigma &= -\frac{2e^2}{3}\int \tau_k(\xi_k) v_k^2 \frac{\partial f}{\partial \xi_k}\rho(\xi_k)\mathrm{d}\xi_k \\ &\simeq \frac{2e^2}{3}v_F^2\rho(0)\frac{16}{3\pi n_i J^2\rho^\alpha(0)}(1-|J|K(0)) \\ &\simeq \frac{2e^2}{3}v_F^2\rho_F\frac{16}{3\pi n_i|J|}\frac{\rho_F}{\rho^\alpha(0)}\Big[\log\left(\frac{T}{T_K^0}\right) \\ &+ \frac{7\zeta(3)}{4\pi^2}\left(\frac{2\alpha k_F}{k_B T}\right)^2\Big].\end{aligned} \tag{74}$$

We have the term $(\alpha/T)^2$ that comes from the spin-orbit interaction, and this term will conceal the logarithmic dependence of the resistivity. Then, the electrical resistivity R in the high temperature region $T \gg T_K^0$ is

$$\begin{aligned}R &= R_0 \\ &\times \left[1-|J|\rho_F\log\left(\frac{2e^\gamma D}{\pi k_B T}\right)+\frac{7\zeta(3)}{4\pi^2}|J|\rho_F\left(\frac{2\alpha k_F}{k_B T}\right)^2\right]^{-1}\end{aligned} \tag{75}$$

where R_0 is a constant. If the term $(\alpha/k_B T)^2$ is larger than the logarithmic term, the resistivity even shows $R \sim T^2$. Hence, the spin-orbit coupling may change the temperature dependence of the resistivity drastically.

VII. STRONG SPIN-ORBIT COUPLING CASE

In this section let us consider the case with strong spin-orbit coupling. For this purpose, we diagonalize the Hamiltonian H_0:

$$\begin{aligned}H_0 &= \sum_{\mathbf{k}}(c^\dagger_{\mathbf{k}\uparrow}c^\dagger_{\mathbf{k}\downarrow})\begin{pmatrix}\xi_{\mathbf{k}} & \alpha(ik_x+k_y)\\ \alpha(-ik_x+k_y) & \xi_{\mathbf{k}}\end{pmatrix}\begin{pmatrix}c_{\mathbf{k}\uparrow}\\ c_{\mathbf{k}\downarrow}\end{pmatrix} \\ &= \sum_{\mathbf{k}}\Big[(\xi_{\mathbf{k}}-\alpha k_\perp)\alpha^\dagger_{\mathbf{k}}\alpha_{\mathbf{k}}+(\xi_{\mathbf{k}}+\alpha k_\perp)\beta^\dagger_{\mathbf{k}}\beta_{\mathbf{k}}\Big],\end{aligned} \tag{76}$$

where $k_\perp = \sqrt{k_x^2+k_y^2}$, and $\alpha_{\mathbf{k}}$ and $\beta_{\mathbf{k}}$ are defined by

$$\alpha_{\mathbf{k}} = u_{\mathbf{k}}c_{\mathbf{k}\uparrow}+v_{\mathbf{k}}c_{\mathbf{k}\downarrow}, \tag{77}$$

$$\beta_{\mathbf{k}} = -v^*_{\mathbf{k}}c_{\mathbf{k}\uparrow}+u_{\mathbf{k}}c_{\mathbf{k}\downarrow}. \tag{78}$$

The coefficients $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ are

$$u_{\mathbf{k}} = \frac{1}{2}, \quad v_{\mathbf{k}} = -\frac{ik_x + k_y}{\sqrt{2}k_\perp}, \tag{79}$$

satisfying $u_{\mathbf{k}}^2 + |v_{\mathbf{k}}|^2 = 1$. We consider the case where the band split is so large that we can neglect the upper band. This means that we keep terms that contain α-operators only. In this approximation the s-d interaction term is

$$\begin{aligned} H_{sd}^{\alpha} &= -\frac{J}{2}\frac{1}{N}\sum_{\mathbf{k}\mathbf{k}'}[S_z\{(u_k u_{k'} - v_k v_{k'}^*)\alpha_{\mathbf{k}}^\dagger \alpha_{\mathbf{k}'} \\ &+ S_+ v_k u_{k'} \alpha_{\mathbf{k}}^\dagger \alpha_{\mathbf{k}'} + S_- u_k v_{k'}^* \alpha_{\mathbf{k}}^\dagger \alpha_{\mathbf{k}'}]. \end{aligned} \tag{80}$$

This is the model of one-channel conduction-electron band that interacts with the localized spin.

Let us consider the following Green's function:

$$G_{\mathbf{k}\mathbf{k}'}^{\alpha}(\tau) = -\langle T_\tau \alpha_{\mathbf{k}}(\tau)\alpha_{\mathbf{k}'}^\dagger(0)\rangle. \tag{81}$$

By using the same method in previous sections, $G_{kk'}^{\alpha}$ is shown to be

$$G_{\mathbf{k}\mathbf{k}'}^{\alpha}(z) = \frac{\delta_{\mathbf{k}\mathbf{k}'}}{z - \xi_{\mathbf{k}\alpha}} + \frac{J}{2N}\frac{\frac{1}{2} + v_{\mathbf{k}} v_{\mathbf{k}'}^*}{(z - \xi_{\mathbf{k}\alpha})(z - \xi_{\mathbf{k}'\alpha})} t(z), \tag{82}$$

for arbitrary complex number z where we defined

$$t(z) = \frac{3J}{16}\frac{F_\alpha(z)}{1 + \frac{J}{2}G_\alpha(z) - \frac{3}{16}\left(\frac{J}{2}\right)^2 F_\alpha(z)^2}, \tag{83}$$

$$F_\alpha(z) = \frac{1}{N}\sum_{\mathbf{k}} \frac{1}{z - \xi_{\mathbf{k}\alpha}}, \tag{84}$$

$$G_\alpha(z) = \frac{1}{N}\sum_{\mathbf{k}} \frac{\bar{n}_{\mathbf{k}\alpha} - 1/2}{z - \xi_{\mathbf{k}\alpha}}. \tag{85}$$

We derive this formula in Appendix. The Kondo temperature T_K^{α} is determined from a singularity of $t(z)$ in the same way as previous sections. We obtain

$$k_B T_K^{\alpha} = \frac{2e^{\gamma}D}{\pi}\exp\left(-\frac{2}{|J|\rho_F}\right). \tag{86}$$

The characteristic energy T_K^{α} is reduced significantly compared to the conventional Kondo temperature by factor 2 in the exponential function. This factor appears because the number of channel of the conduction electrons in this case is just half of the normal Kondo system. The resistivity is also calculated as

$$R = R_0\Big[1 + \frac{\rho_F|J|}{2}\log\left(\frac{2e^{\gamma}D}{\pi k_B T}\right) + \cdots\Big], \tag{87}$$

with a factor 1/2.

VIII. DISCUSSION

We investigated the Kondo effect in the presence of spin-orbit coupling. The influence of band splitting was examined by using the Green's function method where we adopted the decoupling scheme to obtain an approximate solution. The Kondo temperature is reduced by the spin-orbit interaction, and shows a sudden decrease when αk_F is of the order of $k_B T_K^0$. We call this the Kondo collapse due to the spin-orbit coupling. The Kondo effect is suppressed and the $\log T$-dependence of the resistivity will be weakened and concealed. The reduction of T_K as a result of the spin-orbit coupling is consistent with the result for the single-impurity Anderson model using the numerical renormalization group technique[13]. In their work the Kondo temperature is a decreasing function of the Rashba energy $E_R \propto \alpha^2$ when the level of the localized electrons is lowered, that is, in the Kondo region, while it is a increasing function when the localized level is not deep. The variation of the Kondo temperature is approximately linear as a function of E_R, namely, quadratic in α. This is consistent with our result which shows a small variation of the Kondo temperature with the quadratic correction when α is small. In a recent work[12], the Kondo temperature is increased in the presence of the Dzyaloshinski-Moriya interaction. The Dzyaloshinsky-Moriya interaction, however, vanishes in the Kondo region with particle-hole symmetry $\epsilon_d = -U/2$. Hence the result in ref.[12] seems consistent with the result for the s-d model.

As a limit of strong spin-orbit interaction, we can investigate a crossover to a one-channel Kondo problem. The Kondo problem with the spin-orbit coupling is closely related to a multi-channel Kondo problem. The Kondo temperature is reduced considerably because the degeneracy of the conducting electrons becomes half of the conventional Kondo system in this limit. The specific heat also exhibits a $\log T$-term in the present model with one-channel conduction band, and this term appears in the fifth-order of ρJ. This agrees with the original Kondo problem.

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IX. APPENDIX

In this appendix we derive the equation of motion for Green's functions for the single-band s-d model and discuss its physical properties.

A. Green's functions

H_0 was diagonalized by $\alpha_{\mathbf{k}}$ and $\beta_{\mathbf{k}}$:

$$\alpha_{\mathbf{k}} = u_{\mathbf{k}} c_{\mathbf{k}\uparrow} + v_{\mathbf{k}} c_{\mathbf{k}\downarrow}, \tag{88}$$

$$\beta_{\mathbf{k}} = -v_{\mathbf{k}}^* c_{\mathbf{k}\uparrow} + u_{\mathbf{k}} c_{\mathbf{k}\downarrow}. \tag{89}$$

The coefficients $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ are

$$u_{\mathbf{k}} = \frac{1}{2}, \quad v_{\mathbf{k}} = -\frac{ik_x + k_y}{\sqrt{2}k_\perp}, \tag{90}$$

satisfying $u_{\mathbf{k}}^2 + |v_{\mathbf{k}}|^2 = 1$. The s-d interaction part becomes

$$\begin{aligned} H_{sd} &= -\frac{J}{2}\frac{1}{N}\sum_{\mathbf{kk'}}[S_z\{(u_k u_{k'} - v_k v_{k'}^*)\alpha_k^\dagger \alpha_{k'} \\ &- (u_k u_{k'} - v_k^* v_{k'})\beta_{\mathbf{k}}^\dagger \beta_{\mathbf{k'}} \\ &- (u_k v_{k'} + v_k u_{k'})\alpha_{\mathbf{k}}^\dagger \beta_{\mathbf{k'}} - (u_k v_{k'}^* + v_k^* u_{k'})\beta_{\mathbf{k}}^\dagger \alpha_{\mathbf{k'}}\} \\ &+ S_+(v_k u_{k'}\alpha_{\mathbf{k}}^\dagger \alpha_{\mathbf{k'}} - u_k v_{k'}\beta_{\mathbf{k}}^\dagger \beta_{\mathbf{k'}} - v_k v_{k'}\alpha_{\mathbf{k}}^\dagger \beta_{\mathbf{k'}} \\ &+ u_k u_{k'}\beta_{\mathbf{k}}^\dagger \alpha_{\mathbf{k'}}) \\ &+ S_-(u_k v_{k'}^* \alpha_{\mathbf{k}}^\dagger \alpha_{\mathbf{k'}} - v_k^* u_{k'}\beta_{\mathbf{k}}^\dagger \beta_{\mathbf{k'}} + u_k u_{k'}\alpha_{\mathbf{k}}^\dagger \beta_{\mathbf{k'}} \\ &- v_k^* v_{k'}\beta_{\mathbf{k}}^\dagger \alpha_{\mathbf{k'}})]. \end{aligned} \tag{91}$$

The single-band s-d model contains only the following s-d interaction,

$$\begin{aligned} H_{sd}^\alpha &= -\frac{J}{2}\frac{1}{N}\sum_{\mathbf{kk'}}[S_z\{(u_k u_{k'} - v_k v_{k'}^*)\alpha_k^\dagger \alpha_{k'} \\ &+ S_+ v_k u_{k'}\alpha_k^\dagger \alpha_{k'} + S_- u_k v_{k'}^* \alpha_k^\dagger \alpha_{k'}]. \end{aligned} \tag{92}$$

We consider the following Green's functions:

$$\begin{aligned} G_{\mathbf{kk'}}^\alpha(\tau) &= -\langle T_\tau \alpha_{\mathbf{k}}(\tau)\alpha_{\mathbf{k'}}^\dagger(0)\rangle, & (93)\\ \langle\langle S_z\alpha_{\mathbf{k}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_\tau &= -\langle T_\tau (S_z\alpha_{\mathbf{k}})(\tau)\alpha_{\mathbf{k'}}^\dagger(0)\rangle, & (94)\\ \langle\langle S_+\alpha_{\mathbf{k}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_\tau &= -\langle T_\tau (S_+\alpha_{\mathbf{k}})(\tau)\alpha_{\mathbf{k'}}^\dagger(0)\rangle. & (95) \end{aligned}$$

The Fourier transforms are defined similarly:

$$\begin{aligned} G_{\mathbf{kk'}}^\alpha(\tau) &= \frac{1}{\beta}\sum_{\omega_n} e^{i\omega_n\tau} G_{\mathbf{kk'}}^\alpha(i\omega_n), & (96)\\ \langle\langle S_z\alpha_{\mathbf{k}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_\tau &= \frac{1}{\beta}\sum_{\omega_n} e^{i\omega_n\tau}\langle\langle S_z\alpha_{\mathbf{k}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_\omega, & (97)\\ \langle\langle S_+\alpha_{\mathbf{k}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_\tau &= \frac{1}{\beta}\sum_{\omega_n} e^{i\omega_n\tau}\langle\langle S_+\alpha_{\mathbf{k}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_\omega. & (98) \end{aligned}$$

The equations of motion for these Green's functions are derived as follows.

$$\begin{aligned} i\omega_n G_{\mathbf{kk'}}^\alpha(i\omega_n) &= \delta_{\mathbf{kk'}} + \xi_{\mathbf{k}\alpha} G_{\mathbf{kk'}}^\alpha(i\omega_n) \\ &- \frac{J}{2N}\sum_q [(u_k u_q - v_k v_q^*)\langle\langle S_z\alpha_{\mathbf{q}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n} \\ &+ v_k u_q\langle\langle S_+\alpha_{\mathbf{q}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n} \\ &+ u_k v_q^*\langle\langle S_-\alpha_{\mathbf{q}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n}], \end{aligned} \tag{99}$$

$$\begin{aligned} & i\omega_n\langle\langle S_z\alpha_{\mathbf{k}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n} = \xi_{\mathbf{k}\alpha}\langle\langle S_z\alpha_{\mathbf{k}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n} \\ &+ \frac{J}{2N}\sum_q [-(u_k u_q - v_k v_q^*)\langle\langle S_z^2\alpha_{\mathbf{q}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n} \\ &+ v_k u_q^*\langle\langle S_+(n_{\mathbf{k}\alpha} - \frac{1}{2})\alpha_{\mathbf{q}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n} \\ &- u_k v_q^*\langle\langle S_-(n_{\mathbf{k}\alpha} - \frac{1}{2})\alpha_{\mathbf{q}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n}], \end{aligned} \tag{100}$$

$$\begin{aligned} & i\omega_n\langle\langle S_+\alpha_{\mathbf{k}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n} = \xi_{\mathbf{k}\alpha}\langle\langle S_+\alpha_{\mathbf{k}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n} \\ &+ \frac{J}{2N}\sum_{\mathbf{q}} [-(u_k u_q - v_k v_q^*)\langle\langle S_+(n_{\mathbf{k}\alpha} - \frac{1}{2})\alpha_{\mathbf{q}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n} \\ &+ 2u_k v_q^*\langle\langle S_z n_{\mathbf{k}\alpha}\alpha_{\mathbf{q}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n} - u_k v_q^*\langle\langle S_+S_-\alpha_{\mathbf{q}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n}], \end{aligned} \tag{101}$$

$$\begin{aligned} & i\omega_n\langle\langle S_-\alpha_{\mathbf{k}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n} = \xi_{\mathbf{k}\alpha}\langle\langle S_+\alpha_{\mathbf{k}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n} \\ &+ \frac{J}{2N}\sum_{\mathbf{q}} [(u_k u_q - v_k v_q^*)\langle\langle S_-(n_{\mathbf{k}\alpha} - \frac{1}{2})\alpha_{\mathbf{q}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n} \\ &- 2v_k u_q\langle\langle S_z n_{\mathbf{k}\alpha}\alpha_{\mathbf{q}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n} - v_k u_q\langle\langle S_-S_+\alpha_{\mathbf{q}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n}], \end{aligned} \tag{102}$$

where $n_{\mathbf{k}\alpha} = \alpha_{\mathbf{k}}^\dagger \alpha_{\mathbf{k}}$. We have unknown functions $\langle\langle S_a n_{\mathbf{k}\alpha}\alpha_{\mathbf{q}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle$ for $a = z$, $+$ and $-$.

B. Approximate Solution

To obtain a consistent solution, we adopt the following approximation:

$$\langle\langle S_a n_{\mathbf{k}\alpha}\alpha_{\mathbf{q}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle = \langle n_{\mathbf{k}\alpha}\rangle\langle\langle S_a\alpha_{\mathbf{q}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle. \tag{103}$$

Using this approximation and the relation $S_+S_- = 3/4 + S_z - S_z^2$, we obtain

$$\begin{aligned} & (i\omega_n - \xi_{\mathbf{k}\alpha})G_{\mathbf{kk'}}^\alpha(i\omega_n) = \delta_{\mathbf{kk'}} \\ &+ \left(\frac{J}{2N}\right)^2 \sum_{\mathbf{q}} \frac{\bar{n}_{\mathbf{q}} - 1/2}{i\omega_n - \xi_{\mathbf{q}}} \sum_{\mathbf{q'}} \Big[v_k u_{q'}\langle\langle S_+\alpha_{\mathbf{q'}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n} \\ &+ u_k v_{q'}^*\langle\langle S_-\alpha_{\mathbf{q'}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n} \\ &+ (u_k u_{q'} - v_k v_{q'}^*)\langle\langle S_z\alpha_{\mathbf{q'}}; \alpha_{\mathbf{k'}}^\dagger\rangle\rangle_{i\omega_n}\Big] \\ &+ \frac{3}{8}\left(\frac{J}{2N}\right)^2 \sum_{\mathbf{q}} \frac{1}{i\omega_n - \xi_{\mathbf{q}\alpha}} \sum_{\mathbf{q'}} (u_k u_{q'} + v_k v_{q'}^*) G_{\mathbf{q'k'}}^\alpha(i\omega_n), \end{aligned} \tag{104}$$

where $\bar{n}_{\mathbf{q}} = \langle n_{\mathbf{q}\alpha}\rangle$. Here we define

$$\Gamma_{\mathbf{kk'}}(i\omega_n) = \sum_{\mathbf{q}} \Big[(u_k u_q - v_k v_q^*)\langle\langle S_z\alpha_{\mathbf{q}};\alpha^\dagger_{\mathbf{k'}}\rangle\rangle_{i\omega_n} + v_k u_q\langle\langle S_+\alpha_{\mathbf{q}};\alpha^\dagger_{\mathbf{k'}}\rangle\rangle_{i\omega_n} + u_k v_q^*\langle\langle S_-\alpha_{\mathbf{q}};\alpha^\dagger_{\mathbf{k'}}\rangle\rangle_{i\omega_n}\Big]. \tag{105}$$

This quantity reads after substituting the equations for $\langle\langle S_a\alpha_{\mathbf{q}};\alpha^\dagger_{\mathbf{k'}}\rangle\rangle$

$$\begin{aligned}\Gamma_{\mathbf{kk'}} &= \frac{J}{2N}\sum_{\mathbf{q}}\frac{1}{i\omega_n - xi_{\mathbf{k}\alpha}}\sum_{\mathbf{q'}}\Big[\\ &- v_k u_{q'}\left(\bar{n}_{\mathbf{q}}-\frac{1}{2}\right)\langle\langle S_+\alpha_{\mathbf{q'}};\alpha^\dagger_{\mathbf{k'}}\rangle\rangle_{i\omega_n} \\ &- u_k v^*_{q'}\left(\bar{n}_{\mathbf{q}}-\frac{1}{2}\right)\langle\langle S_-\alpha_{\mathbf{q'}};\alpha^\dagger_{\mathbf{k'}}\rangle\rangle_{i\omega_n} \\ &- (u_k u_{q'} - v_k v^*_{q'})\left(\bar{n}_{\mathbf{q}}-\frac{1}{2}\right)\langle\langle S_z\alpha_{\mathbf{q'}};\alpha^\dagger_{\mathbf{k'}}\rangle\rangle_{i\omega_n} \\ &- \frac{3}{8}(u_k u_{q'} + v_k v^*_{q'})G^\alpha_{\mathbf{q'k'}}(i\omega_n)\Big] \\ &= -\frac{J}{2N}\frac{3}{8}\sum_{\mathbf{q}}\frac{\bar{n}_{\mathbf{k}}-1/2}{i\omega_n-\xi_{\mathbf{q}\alpha}}\Gamma_{\mathbf{kk'}} - \frac{J}{2N}\frac{3}{8}\sum_{\mathbf{q}}\frac{1}{i\omega_n-\xi_{\mathbf{q}\alpha}} \\ &\times \sum_{\mathbf{q'}}(u_k u_{q'} + v_k v^*_{q'})G^\alpha_{\mathbf{q'k'}}(i\omega_n).\end{aligned} \tag{106}$$

Then we obtain

$$\begin{aligned}G^\alpha_{\mathbf{kk'}}(i\omega_n) &= \frac{\delta_{\mathbf{kk'}}}{i\omega_n-\xi_{\mathbf{k}\alpha}} + \frac{3}{8}\left(\frac{J}{2N}\right)^2\frac{1}{i\omega_n-\xi_{\mathbf{k}\alpha}} \\ &\times \sum_{\mathbf{q'}}\frac{1}{i\omega_n-\xi_{\mathbf{q'}\alpha}}\frac{1}{1+\frac{J}{2N}\sum_{\mathbf{p}}\frac{\bar{n}_{\mathbf{p}}-1/2}{i\omega_n-\xi_{\mathbf{p}\alpha}}} \\ &\times \sum_{\mathbf{q}}(u_k u_q + v_k v_q^*)G^\alpha_{\mathbf{qk'}}(i\omega_n).\end{aligned} \tag{107}$$

We have set $u_{\mathbf{k}} = 1/\sqrt{2}$. Because $v_{\mathbf{k}}$ satisfies $v_{\mathbf{k}} = -v_{-\mathbf{k}}$ and $|v_{\mathbf{k}}|^2 = 1/2$, we have

$$\begin{aligned}\sum_{\mathbf{k}} v^*_{\mathbf{k}}G^\alpha_{\mathbf{kk'}}(i\omega_n) &= \frac{v^*_{\mathbf{k'}}}{i\omega_n-\xi_{\mathbf{k'}\alpha}}\Big[1-\frac{3}{8}\left(\frac{J}{2}\right)^2\frac{1}{2}F_\alpha(i\omega_n)^2 \\ &\times \frac{1}{1+(J/2)G_\alpha(i\omega_n)}\Big]^{-1},\end{aligned} \tag{108}$$

where we set

$$F_\alpha(z) = \frac{1}{N}\sum_{\mathbf{k}}\frac{1}{z-\xi_{\mathbf{k}\alpha}}, \tag{109}$$

$$G_\alpha(z) = \frac{1}{N}\sum_{\mathbf{k}}\frac{\bar{n}_{\mathbf{k}\alpha}-1/2}{z-\xi_{\mathbf{k}\alpha}}. \tag{110}$$

We define

$$t(z) = \frac{3J}{16}\frac{F_\alpha(z)}{1+\frac{J}{2}G_\alpha(z)-\frac{3}{16}\left(\frac{J}{2}\right)^2F_\alpha(z)^2}. \tag{111}$$

Then $G^\alpha_{\mathbf{kk'}}$ and $\Gamma_{\mathbf{kk'}}$ read

$$G^\alpha_{\mathbf{kk'}}(z) = \frac{\delta_{\mathbf{kk'}}}{z-\xi_{\mathbf{k}\alpha}} + \frac{J}{2N}\frac{\frac{1}{2}+v_k v^*_{k'}}{(z-\xi_{\mathbf{k}\alpha})(z-\xi_{\mathbf{k'}\alpha})}t(z), \tag{112}$$

$$\Gamma_{\mathbf{kk'}}(z) = -\frac{\frac{1}{2}+v_k v^*_{k'}}{z-\xi_{\mathbf{k'}\alpha}}t(z), \tag{113}$$

for arbitrary complex number z. The Kondo temperature T_K^α is determined from a singularity of $t(z)$ in the same way as previous sections. We obtain

$$T_K^\alpha = \frac{2e^\gamma D}{\pi}\exp\left(-\frac{2}{|J|\rho_F}\right). \tag{114}$$

The characteristic energy T_K^α is reduced significantly compared to the conventional Kondo temperature by factor 2 in the exponential function:

$$T_K^\alpha \sim \left(\frac{T_K^0}{D}\right)T_K^0. \tag{115}$$

This factor appears because the number of channel of the conduction electrons in this case is just half of the normal Kondo system. The resistivity is also calculated as

$$R = R_0\Big[1+\frac{\rho_F|J|}{2}\log\left(\frac{2e^\gamma D}{\pi k_B T}\right)+\cdots\Big], \tag{116}$$

with a factor 1/2.

C. Entropy and Specific Heat

The energy expectation value $E = \langle H\rangle$ is given by

$$\begin{aligned}E &= \sum_{\mathbf{k}}\xi_{\mathbf{k}}\langle\alpha^\dagger_{\mathbf{k}}\alpha_{\mathbf{k}}\rangle - \frac{J}{2N}\sum_{\mathbf{kk'}}\langle\{S_z(u_k u_{k'} - v_k v^*_{k'}) \\ &+ S_+ v_k u_{k'} + S_- u_k v^*_{k'}\}\alpha^\dagger_{\mathbf{k}}\alpha_{\mathbf{k'}}\rangle \\ &= \frac{1}{\beta}\sum_{\mathbf{k}\omega_n}\xi_{\mathbf{k}\alpha}G^\dagger_{\mathbf{kk}}(i\omega_n) - \frac{J}{2}\frac{1}{\beta N}\sum_{\mathbf{k}}\Gamma_{\mathbf{kk}}(i\omega_n) \\ &= \frac{1}{\beta}\sum_{\mathbf{k}\omega_n}\frac{\xi_{\mathbf{k}\alpha}}{i\omega_n-\xi_{\mathbf{k}\alpha}} + \frac{J}{2}\frac{1}{\beta N}\sum_{\mathbf{k}\omega_n}\frac{i\omega_n t(i\omega_n)}{(i\omega_n-\xi_{\mathbf{k}\alpha})^2}.\end{aligned} \tag{117}$$

The expectation value of the interaction Hamiltonian is denoted as V. V is given by

$$\begin{aligned}V &= -\frac{J}{2N}\sum_{\mathbf{kk'}}\langle\{S_z(u_k u_{k'} - v_k v^*_{k'}) + S_+ v_k u_{k'} \\ &+ S_- u_k v^*_{k'}\}\alpha^\dagger_{\mathbf{k}}\alpha_{\mathbf{k'}}\rangle \\ &= -\frac{J}{2}\frac{1}{\beta N}\sum_{\mathbf{k}}\Gamma_{\mathbf{kk}}(i\omega_n) = \frac{J}{2}\frac{1}{\beta N}\sum_{\mathbf{k}\omega_n}\frac{t(i\omega_n)}{i\omega_n-\xi_{\mathbf{k}\alpha}}.\end{aligned} \tag{118}$$

This is written as

$$V = \frac{J}{2}\rho(0)\mathrm{Re}\int_{-D}^{D} d\omega f(\omega)t(\omega - i\delta), \quad (119)$$

where we adopted the approximation

$$F_\alpha(\omega \pm i\delta) = \mp\pi\rho(0)i. \quad (120)$$

$\rho(\omega)$ is the density of states of conduction electrons.

We need $t(z)$ to estimate V. $G_\alpha(z)$, which appears in the denominator of $t(z)$, contains a singularity. $G_\alpha(z)$ is written as

$$G_\alpha(z) = R_\alpha(z) + \frac{J}{2N}\frac{1}{\beta N}\sum_{\mathbf{k}\omega_n}\frac{1}{z-\xi_{\mathbf{k}\alpha}}\frac{t(i\omega_n)}{(i\omega_n - \xi_{\mathbf{k}\alpha})^2}, \quad (121)$$

where

$$R_\alpha(z) = \frac{1}{\beta}\sum_{\omega_n}\frac{F_\alpha(i\omega_n) - F_\alpha(z)}{z - i\omega_n} - \frac{1}{2}F_\alpha(z). \quad (122)$$

$R_\alpha(z)$ is evaluated as[18]

$$R_\alpha(\omega - i\delta) \approx \rho(0)\left[\psi\left(\frac{1}{2}+\frac{\beta D}{2\pi}\right) - \psi\left(\frac{1}{2}+i\frac{\beta z}{2\pi}\right)\right], \quad (123)$$

where ψ is the digamma function and D is the cutoff energy. We use the following relation,

$$\begin{aligned} &1 + \frac{\rho(0)J}{2}\Big[\log\left(\frac{D}{2\pi k_B T}\right) - \psi\left(\frac{1}{2}+i\frac{\beta\omega}{2\pi}\right)\Big] \\ &= \frac{\rho(0)J}{2}\Big[\log\frac{T_K^\alpha}{T} - g(\beta\omega)\Big], \end{aligned} \quad (124)$$

where

$$g(x) = \psi\left(\frac{1}{2}+i\frac{x}{2\pi}\right) - \psi\left(\frac{1}{2}\right). \quad (125)$$

Then the interaction energy is

$$V = -\frac{3\pi}{16}\rho(0)J\mathrm{Im}\int_{-D}^{D} d\omega f(\omega)\frac{1}{\log(T_K^\alpha/T) - g(\beta\omega)}, \quad (126)$$

where we neglected the term of the order of $(\rho(0)J)^2$ in the denominator of $t(z)$. V has a logarithmic temperature dependence. Because of the relation between the free energy and V,

$$V = J\frac{\partial F}{\partial J}, \quad (127)$$

the additional entropy $\Delta S(T)$ is

$$\Delta S(T) = -\frac{\partial}{\partial T}(F - F_0) = -\int_0^J \frac{dJ'}{J'}\frac{\partial}{\partial T}V(J', T). \quad (128)$$

To estimate V, we use the expansion formula for the Fermi distribution function $f(\omega)$:

$$\int_{-D}^{D} d\omega f(\omega)h(\omega) = \int_{-D}^{0} d\omega h(\omega) + \frac{\pi^2}{6}(k_B T)^2 h'(0), \quad (129)$$

for a differentiable function $h(\omega)$. Using this, we obtain

$$\begin{aligned} V &= -\frac{3\pi}{16}\rho(0)J\mathrm{Im}\int_{-D}^{0} d\omega\frac{1}{\log(T_K^\alpha/T) - g(\beta\omega)} \\ &- \frac{3\pi}{16}\frac{\pi^2}{6}(k_B T)^2\rho(0)J\mathrm{Im}\frac{\partial}{\partial\omega}\frac{1}{\log(T_K^\alpha/T) - g(\beta\omega)}\Big|_{\omega=0}. \end{aligned} \quad (130)$$

We are interested in logarithmic terms $\log(D/k_B T)$, $\log(D/k_B T)^2$ and so on in the region $|\log(T_K^\alpha/T)| \gg 1$. The second term is written as

$$V_2 = -\frac{\pi^4}{128}k_B T\rho(0)J\frac{1}{(\log(T_K^\alpha/T))^2}. \quad (131)$$

This is expanded as in terms of $\rho(0)J$:

$$\begin{aligned} V_2 &= \frac{\pi^4}{32}\left(\frac{\rho(0)J}{2}\right)^4 k_B T\log\left(\frac{D}{k_B T}\right) \\ &- \frac{3\pi^4}{64}\left(\frac{\rho(0)J}{2}\right)^5 k_B T\log\left(\frac{D}{k_B T}\right)^2. \end{aligned} \quad (132)$$

In the first term of V, the logarithmic corrections never emerge from the region where $\beta\omega$ is large because we have $g(\beta\omega) \sim \log(\beta\omega)$ for large ω and the T-dependence is canceled with $\log(T_K^\alpha/T)$. When $\beta\omega$ is small, $g(\beta\omega)$ is expressed in a power series of $\beta\omega$. A dominant contribution is of the order of $(\log(T_K^\alpha/T))^{-2}$. The integral is restricted on the interval $(-k_B T, 0)$ and the first term V_1 is estimated as

$$\begin{aligned} V_1 &\simeq -\frac{3\pi}{16}\rho(0)J\frac{1}{(\log(T_K^\alpha/T))^2}\int_{-k_B T}^{0} d\omega\mathrm{Im}\psi\left(\frac{1}{2}+i\frac{\beta\omega}{2\pi}\right) \\ &= -\frac{3\pi}{16}k_B T\rho(0)J\frac{1}{(\log(T_K^\alpha/T))^2}\pi\Big[-\frac{1}{8} + 0.0052 \\ &- 0.00738 + 0.000026\cdots\Big]. \end{aligned} \quad (133)$$

As a result, V is given as

$$V = -\frac{A}{2}k_B T\rho(0)J\frac{1}{(\log(T_K^\alpha/T))^2}, \quad (134)$$

for a constant $A > 0$.

From the relation $T_K^\alpha = D\exp(2/(\rho(0)J))$, we have

$$\frac{d\rho(0)J}{\rho(0)J} = -\frac{1}{\log(T_K^\alpha/D)}d\log T_K^\alpha. \quad (135)$$

Using this formula, the entropy obtained from the interaction energy V is

$$\Delta S = -\frac{\partial\Delta F}{\partial T}, \quad (136)$$

where

$$\Delta F = -k_B A\Big[T\frac{1}{(\log(D/T))^2}\Big(\frac{\rho(0)J}{2}+\frac{1}{\log(T_K^\alpha/T)} - \frac{2}{\log(D/T)}\log\Big|\frac{\rho(0)J}{2}\log\left(\frac{T_K^\alpha}{T}\right)\Big|\Big)\Big], \quad (137)$$

is the free energy. Because of the relation

$$\log\left(\frac{T_K^\alpha}{T}\right) = \frac{2}{\rho(0)J} + \log\left(\frac{2e^\gamma}{\pi}\frac{D}{k_BT}\right), \quad (138)$$

up to the fifth order of $\rho(0)J$, ΔS is given as

$$\begin{aligned}\Delta S &= k_B A\Big[\frac{1}{3}\left(\frac{\rho(0)J}{2}\right)^3 + \frac{1}{2}\left(\frac{\rho(0)J}{2}\right)^4 \\ &- \frac{1}{2}\left(\frac{\rho(0)J}{2}\right)^4\log\left(\frac{D}{k_BT}\right) + \frac{3}{5}\left(\frac{\rho(0)J}{2}\right)^5\left(\log\frac{D}{k_BT}\right)^2 \\ &- \frac{6}{5}\left(\frac{\rho(0)J}{2}\right)^5\log\left(\frac{D}{k_BT}\right)\Big]. \end{aligned} \quad (139)$$

The logarithmic term first appears in the fourth order of $\rho(0)J$. Then the correction to the specific heat $\Delta C = T\partial\Delta S/\partial T$ is

$$\frac{\Delta C}{k_B} \simeq \frac{A}{2}\left(\frac{\rho(0)J}{2}\right)^4\Big[1-\frac{12}{5}\left(\frac{\rho(0)J}{2}\right)\log\left(\frac{D}{k_BT}\right)\Big]. \quad (140)$$

Hence the specific heat exhibits a logarithmic behavior at low temperatures. A $\log T$-term appears in the fifth order of $\rho(0)J$; this agrees with the original Kondo problem[2]. In the original Kondo problem, the entropy and the specific heat were evaluated as[2, 19]

$$\Delta S_{sd} \simeq k_B\frac{\pi^2}{4}(\rho J)^3\Big[1-3\rho J\log\left(\frac{D}{k_BT}\right)\Big], \quad (141)$$

$$\Delta C_{sd} \simeq k_B\frac{3\pi^3}{4}(\rho J)^4\Big[1-4\rho J\log\left(\frac{D}{k_BT}\right)\Big]. \quad (142)$$

This suggests that[18]

$$\begin{aligned}\Delta C_{sd} &\simeq k_B\frac{3\pi^3}{4}(\rho J)^4\frac{1}{(1+\rho J\log(D/k_BT)^4} \\ &\simeq k_B\frac{3\pi^3}{4}\frac{1}{(\log(T_K/T))^4}, \end{aligned} \quad (143)$$

as an expansion in terms of $1/\log(T_K/T)$. In the present model, the coefficients are reduced, where 4 is reduced to 12/5 in front of $\rho J\log(D/k_BT)$ in the specific heat compared to the usual s-d model, and the divergence near the Kondo temperature is moderated. Because the formation of a local singlet by the the conduction electrons is weakened in a one-channel case, the entropy decreases more slowly as the temperature is decreased.

In the region $|\log(D/k_BT)| \gg 1$ and $|\log(T_K^\alpha/T)| \gg 1$, ΔS is obtained as a double-power series of $1/\log(D/k_BT)$ and $1/\log(T_K^\alpha/T)$:

$$\Delta S \simeq k_B A\Big[\frac{1}{(\log(D/k_BT))^2}\left(\frac{\rho(0)J}{2}+\frac{1}{\log(T_K^\alpha/T)}\right)\Big]. \quad (144)$$

Then we obtain

$$\Delta C \simeq k_B A\frac{1}{(\log(D/k_BT))^2}\frac{1}{(\log(T_K^\alpha/T))^2}. \quad (145)$$

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